Your textbook has defined an “unusual” event as one that has a very small chance of happening. That “chance”, in most cases, is defined as being 5% or less. You will be using the same idea in this section of the course. The percent used for the definition of “unusual” will be called “α” (alpha, the small Greek letter “a”); this can change depending on the “level” of the “test”; for now, we will say α = 0.05 (5%). In general, hypothesis testing is attempting to find out if a population parameter (mean or proportion) has significantly changed from what you are told it is. Is there a statistical difference?

When you were estimating the population mean, μ, from a set of data, the focus was on the level of confidence you had in your answer as a percent, 90% or 95% or 99%. With hypothesis testing, the focus is on the amount of acceptable error you are willing to tolerate; this is “α” (alpha). Generally, α = 0.05 (5%) or 0.01 (1%).

Let’s say a machine has been making a metal blank or cylinder that is supposed to be three-eighths of an inch in diameter. After a number of days of production, experience has told the company that the machine might drift out of set or parts get worn and the diameter might become larger; the standard deviation of the item stayed consistently the same at 0.0013 inch (σ = 0.0013). Here are some data from samples taken over a two-hour period by a worker grabbing samples and measuring the diameter with a micrometer; (three-eighths inch = 0.375 inch.) These data are approximately normal with no outliers.

<table>
<thead>
<tr>
<th>x</th>
<th>freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.372</td>
<td>1</td>
</tr>
<tr>
<td>0.373</td>
<td>3</td>
</tr>
<tr>
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<tr>
<td>0.375</td>
<td>6</td>
</tr>
<tr>
<td>0.376</td>
<td>9</td>
</tr>
<tr>
<td>0.377</td>
<td>7</td>
</tr>
<tr>
<td>0.378</td>
<td>3</td>
</tr>
</tbody>
</table>

What is the mean? What is n? If the company wants the mean to be 0.375, how “unusual” is it for a sample of this size to have the sample mean it has? In order to get this answer, you will be doing a “hypothesis test”. Since the machine was producing blanks with a mean diameter of 0.375 inches, your “null” hypothesis will be μ = 0.375. (“Null” means “zero”; our stating point is at “time zero”.) Since your data indicate that the mean is now greater than 0.375, your alternate hypothesis is μ > 0.375. The company worries about the machine drifting out of set and making defective blanks. Here’s the set up: In general, we assume the machine has not drifted out of set until we have data to “prove” it, so the beginning (null) hypothesis is always what you are told to be true.

\[ H_0: \mu = 0.375 \]

\[ H_1 \text{ or } H_a: \mu > 0.375 \]

How unusual is the mean? Find a Z value for the mean and the area of the normal curve that is above that z score.

\[ Z = \frac{0.37553 - 0.375}{0.0013} = 2.37 \]

\[ P(z > 2.37) = \text{normalcdf}(2.38, 100) = 0.00866 \]

In this case, 0.00866, called the p-value, the probability of that event, is < 0.05, a number that is used to decide if something is “unusual”. α = 0.05 is also known as the level of significance of the test.

**Conclusion:** Reject the null hypothesis since the p-value is < 0.05. This machine has drifted out of set and will need to be reset. (The conclusion does not say, with confidence, that the new mean is 0.37553.) Your “conclusion” should be also stated in the context of the problem. These results are called statistically significant.

**NOTE:** In reality, since there is a “tolerance” in the production of this blank, that is, it is okay that some of them are slightly larger than 0.0375 inches, the value that will be used for the definition of “unusual”, α, in this case, will be 0.01 (α = 0.01). The p-value was lower than 0.01, so this machine needs to be reset. The worker will not be doing all the calculations; as the data are collected, it will be put into a computer at the work site programmed to do the calculations. Shutting down production, even temporarily, is serious, so the computer will actually give the decision on the screen and will most likely automatically e-mail a supervisor about the results.

Sample mean = 0.37553; n = 34
Can this be done on the calculator? Of course it can! Note that the population standard deviation is assumed to be $\sigma = 0.0013$, so you will be doing a Z-Test; find it on your calculator and fill in the necessary; remember, the definition of “unusual” will be called “α”. For your “test”, you want “$> \mu_0$” because that was your alternate hypothesis. What did you get? See below. When using the calculator, you still need to write your two hypotheses and conclusion; on a test, you may have to show the set-up using the correct formula.

What’s a T-Test? As you may remember from confidence intervals, the t distribution and degrees of freedom were used when you had (1) a small set of data and/or (2) the population standard deviation, $\sigma$, was unknown. In like manner, in the previous problem, if $\sigma$ was not known to be 0.0013, $S_x$, the sample standard deviation, would have to be used. The calculator will calculate the sample standard deviation and find the proper degrees of freedom, when you choose to use a T-Test. Let’s do it with this data: d.f. is 33, the sample mean is the same and the sample standard deviation will be used. The calculator set up looks like this: T-Test: Data $\mu_0$: 0.375 List: Put in where ever you put the data Freq: Where ever you put the frequencies Choose “$> \mu_0$” Calculate ENTER. What did you get? Notice in this case, the t value is considerably less than the z value for a Z-Test and the p-value is not less than $\alpha = 0.01$. That’s because the t distribution is a “flatten” normal distribution spreading out the area under the curve. The decision to use one alpha value, 0.05 or 0.01, over the other is entirely a decision of the company.

Try this problem using the calculator. Complete the parts that are requested. The average monthly sales for a salesperson at a company is $47,526; salespeople earn 8% commission. Rena has been with the company for 7 years and thinks her sales are significantly higher than the average. Here are her sales summary for the last year. $\bar{x} = $48318; $S_x = $1497 What is n? What are your hypotheses? What formula will you use? Why did you use that formula (z or t)? What value did you get? Look at the distribution in the book to estimate the p-value. Fill in the formulas and use the calculator to find the results. What’s the p-value? Let’s use $\alpha = 0.05$. What’s your conclusion? Write it in the context of the problem.

Z-Test $\mu > .375$ $z = 2.374595794$ $p = .0087840758$ $\bar{x} = .3755294118$ $S_x = .0015615706$ n = 34

Notice the mathematical results are about the same as what you got doing it long-hand.

T-Test $\mu > .375$ $t = 1.976839583$ $p = .0282325506$ $\bar{x} = .3755294118$ $S_x = .0015615706$ n = 34

Sample problem: n = 12; d.f. = 11; $H_0: \mu = $47,526 $H_1$ or $H_a: \mu > $47,526

Since the population standard deviation is not known, use the t-distribution.

$$t = \frac{(\bar{x} - \mu)}{\frac{s_x}{\sqrt{n}}} = \frac{(48318 - 47526)}{(1497 / \sqrt{12})} \approx 1.833$$

At row 11 for the d.f., 1.833 is between 0.05 and 0.025, so the p-value is between those numbers, namely, less than $\alpha = 0.05$: reject $H_0$.

Calculator results: $t = 1.832711075$, $p = 0.0470$

CONCLUSION: Since the p-value is less than $\alpha = 0.05$, reject $H_0$. The data indicate that Rena’s average sales are significantly larger than the company average sales. (There is a statistically significant difference in the mean sales of Rena and other salespersons.)